Resummation effects in Higgs boson transverse momentum distribution within the framework of unintegrated parton distributions

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Abstract

The cross sections describing the transverse momentum distributions of Higgs bosons are discussed within the framework of unintegrated parton distributions obtained from the CCFM equations in the single loop approximation. It is shown how the approximate treatment of the CCFM equations generates the standard expressions describing the soft gluon resummation effects in the corresponding cross sections. Possible differences between exact and approximate solutions of the CCFM equations are discussed on the example of the $gg \rightarrow H$ fusion mechanism, which gives the dominant contribution to Higgs production.

1 Introduction

The transverse momentum distributions of Drell-Yan pairs, the W or Z_0 gauge bosons or the Higgs bosons produced in high energy hadronic collisions are known to be strongly affected by the soft gluon (recoil) resummation effects [1] - [17]. These effects are particular important in the region $p_T^2 \ll Q^2$, where Q and p_T denote the mass and transverse momentum of the virtual photon γ^* or the W, Z_0 or Higgs bosons, respectively. The soft gluon resummation effects are also important in the prompt photon production in hadronic collisions.

All these reactions reflect the basic partonic fusion subprocesses, i.e. $q + \bar{q} \rightarrow \gamma^* \rightarrow l^+ l^-$, $q + \bar{q} \rightarrow W$, $q + \bar{q} \rightarrow Z_0$, $g + g \rightarrow H$, $g + q(\bar{q}) \rightarrow q(\bar{q}) + \gamma$ etc. The transverse momenta of the lepton pairs, the W, Z_0 and/or Higgs bosons etc. should therefore just be equal to the sum of the transverse momenta of the colliding partons. The transverse momentum distributions of the lepton pairs, the W, Z_0 and/or Higgs bosons etc. produced through the fusion of partons do therefore probe the transverse momentum distributions of the partons, i.e. the *unintegrated* parton distributions [3, 4],[18]-[27]. The corresponding cross sections are determined by the k_T factorisation - like convolutions (subject to the transverse momentum conservation) of the unintegrated parton distributions with the corresponding partonic cross sections. In this simple extension of the collinear picture to the more exclusive configuration, in which the transverse momenta are not integrated over the gluon resummation effects, are naturally attributed to the unintegrated parton distributions. Evolution of the unintegrated distributions is described in perturbative QCD by the Catani, Ciafaloni, Fiorani, Marchesini (CCFM) equation [28]. It corresponds to the sum of the ladder diagrams with the angular ordering along the chain. The gluon radiation controlling evolution of the unintegrated parton distributions generates the transverse momentum of the partons through the simple recoil effect. The CCFM equation interpolates between the (LO) DGLAP evolution at large and moderately small values of x and the BFKL dynamics at low x. Pure DGLAP evolution corresponds to the so called "single loop approximation" of the CCFM evolution [28]-[32].

The purpose of this paper is to examine the transverse momentum distributions of the Higgs bosons produced in hadronic collisions using the unintegrated parton distributions framework. To be precise we shall limit ourselves to the $gg \rightarrow H$ fusion mechanism and determine the corresponding cross sections in terms of the unintegrated gluon distributions. The latter will be obtained from the solution of the CCFM equation in the single loop approximation, which may be adequate for the moderately small values of x probed in the (central) Higgs boson production. We shall utilise the fact that this equation can be diagonalised by the Fourier–Bessel transform and becomes evolution equation for the (scale dependent) parton distributions $\bar{f}_i(x, b, Q)$ for fixed b, where b is a vector of impact parameter conjugate to the transverse momentum. We express the cross sections in terms of the function $f_G(x_{1,2}, b, Q)$ and show that the CCFM equation correctly reproduces the double and single $\ln(Q^2 b^2)$ resummation in the limit $Q^2 b^2 \gg 1$, that corresponds to $p_T^2 \ll Q^2$.

The content of our paper is as follows. In the next section we recall the basic formulas describing the soft gluon resummation effects in the transverse momentum distributions de-

scribing of Higgs bosons produced through the gluon-gluon fusion mechanism. In section 3 we discuss this distribution within the unintegrated gluon distributions framework. We point out that the basic quantities controlling the transverse momentum distributions in this framework are the unintegrated distributions in the $b-$representation. In section 4 we discuss the CCFM equation in this representation. Section 5 contains discussion of the soft gluon resummation formulas as the result of the approximate treatment of the solution(s) of the CCFM equation in the $b-$representation. In section 6 we present numerical results concerning the transverse momentum distributions of Higgs bosons and compare predictions based upon exact solution of the CCFM equation with the standard LO soft gluon resummation formulas. Finally, in section 7 we give summary of our results.

2 Soft gluon resummation formulas

The soft gluon resummation effects are most conveniently formulated using the b-representation. Thus the cross-section for Higgs boson production in $p\bar{p}$ collisions corresponding to the gluon - gluon fusion subprocess $gg \to H$ is expressed in the following way [14, 17]

$$\frac{\partial \sigma}{\partial Q^2 \partial y \partial p_T^2} = \frac{\sigma_0 Q^2}{\tau s} \pi^2 \delta(Q^2 - m_H^2) \frac{1}{2} \int_0^\infty db \, b \, J_0(b p_T) \, W_{GG}(x_1, x_2, b, Q), \tag{1}$$

with $J_0(z)$ being the Bessel function. In eq. (1), p_T and y denote the transverse momentum and rapidity of the Higgs boson, respectively, and m_H is the Higgs boson mass while $x_{1,2}$ are the longitudinal momentum fractions of the gluons given by

$$x_1 = \sqrt{\frac{Q^2 + p_T^2}{s}} \exp(y)$$

$$x_2 = \sqrt{\frac{Q^2 + p_T^2}{s}} \exp(-y) \tag{2}$$

where s is the square of the CM energy of the colliding $p\bar{p}$ system, and

$$\tau = x_1 x_2. \tag{3}$$

The cross section σ_0 is given by:

$$\sigma_0 = \frac{\sqrt{2} \, G_F}{576 \, \pi} \alpha_S(\mu_r^2) \tag{4}$$

where G_F is the Fermi constant. The corresponding cross-section for the Drell-Yan production of muon pairs reads:

$$\frac{\partial \sigma}{\partial Q^2 \partial y \partial p_T^2} = \frac{\sigma_0^{DY}}{\tau} \frac{1}{2} \int db \, b \, J_0(b p_T) \sum_i e_i^2 \left\{ W_{\bar{q}_i q_i}(x_1, x_2, b, Q) + W_{q_i \bar{q}_i}(x_1, x_2, b, Q) \right\} \tag{5}$$

where now p_T, Q and y denote the transverse momentum, invariant mass and rapidity of the lepton pair, respectively, and $x_{1,2}$ denote the momentum fractions of the parent hadrons carried by a quark or antiquark and are defined by eq. (2). The elementary DY cross section σ_0^{DY} is given by:

$$\sigma_0^{DY} = \frac{4\pi\alpha_{em}^2}{9sQ^2}. \tag{6}$$

The basic functions driving the corresponding cross sections are the functions $W_{ij}(x_1, x_2, b, Q)$ which can be interpreted as the scale dependent parton luminosities in the $b-$representation. The soft gluon resummation gives the following representation of these functions

$$W_{ij}(x_1, x_2, b, Q) = W_{ij}^{NP}(x_1, x_2, b, Q) \exp\left\{\frac{1}{2}\left[S_i(b, Q) + S_j(b, Q)\right]\right\}$$
$$\times\ x_1\, p_i^{eff}(x_1, c^2/b^2)\, x_2\, p_j^{eff}(x_2, c^2/b^2). \tag{7}$$

The functions p_i^{eff} are given by the conventional integrated parton distributions p_i probed at the scale $\mu^2 = c^2/b^2$ with $c \sim 1$ [6], i.e.

$$p_i^{eff}(x, \mu^2) = p_i(x, \mu^2) + O(\alpha_s(\mu^2)) \tag{8}$$

with the $O(\alpha_s(\mu^2))$ corrections given in [7, 8]. The exponents S_i are given by:

$$S_i(b, Q) = -\int_{c^2/b^2}^{Q^2} \frac{d\bar\mu^2}{\bar\mu^2}\left[\ln\left(\frac{Q^2}{\bar\mu^2}\right) A_i(\alpha_s(\bar\mu^2)) + B_i(\alpha_s(\bar\mu^2))\right], \tag{9}$$

where the functions A_i and B_i are defined by the perturbative expansion:

$$A_i(\alpha_s) = \sum_{n=1}^\infty \left(\frac{\alpha_s}{2\pi}\right)^n A_i^{(n)}$$

$$B_i(\alpha_s) = \sum_{n=1}^\infty \left(\frac{\alpha_s}{2\pi}\right)^n B_i^{(n)}. \tag{10}$$

The LO coefficients are given by [9]:

$$A_G^{(1)} = 2C_A$$
$$B_G^{(1)} = \frac{4}{3}N_f T_R - \frac{11}{3}C_A \tag{11}$$
$$A_q^{(1)} = A_{\bar q}^{(1)} = C_F$$
$$B_q^{(1)} = B_{\bar q}^{(1)} = -\frac{3}{2}C_F \tag{12}$$

where for the $SU(3)_c$ we have: $C_A = 3, C_F = 4/3, T_R = 1/2$, and N_f is the number of active flavours. Higher order coefficients are defined in [10]. The factors $W_{ij}^{NP}(x_1, x_2, b, Q)$ describe the non-perturbative contribution(s) [13, 14, 17].

4

Coefficients $A_i^{(n)}$ and $B_i^{(n)}$ are directly connected to the part of the splitting functions $P_{ij}(z)$ corresponding to real emission. To this aim we introduce the functions $R_G(z)$ and $R_q(z)$ defined by:

$$R_G(z) = z[P_{gg}(z) + P_{qg}(z)] \tag{13}$$

$$R_q(z) = z[P_{qq}(z) + P_{gq}(z)], \tag{14}$$

and rearrange these functions as below

$$R_G(z) = \frac{r_G}{1-z} + \bar{R}_G(z) \tag{15}$$

$$R_q(z) = \frac{r_q}{1-z} + \bar{R}_q(z) \tag{16}$$

where $r_q = C_F$ and $r_G = 2C_A$, and $R_G(z)$ and $R_q(z)$ are regular at $z = 1$. In LO we get the following relations

$$A_i^{(1)} = r_i, \qquad\qquad i = q, G \tag{17}$$

$$B_i^{(1)} = 2 \int_0^1 dz\, \bar{R}_i(z). \tag{18}$$

It is also useful to remind the following identity:

$$\int_0^1 dz\, [R_q(z) - P_{qq}(z)] = 0. \tag{19}$$

3 Transverse momentum distributions of Higgs bosons and DY pairs

The cross section for Higgs production in $p\bar{p}$ collisions corresponding to the gluon–gluon fusion subprocess $gg \to H$ can be expressed in the following way in terms of the scale dependent unintegrated gluon distributions $f_G(x_{1,2}, k_T, Q)$

$$\frac{\partial \sigma}{\partial Q^2 \partial y \partial p_T^2} = \frac{\sigma_0 Q^2}{\tau s} \pi \delta(Q^2 - m_H^2)$$

$$\times \int \frac{d^2\mathbf{k_1}\, d^2\mathbf{k_2}}{\pi^2} f_G(x_1, k_1, Q)\, f_G(x_2, k_2, Q)\, \delta^{(2)}(\mathbf{k_1} + \mathbf{k_2} - \mathbf{p_T}). \tag{20}$$

In this equation $\mathbf{p_T}$ and y denote, as before, the transverse momentum and rapidity of the Higgs boson, respectively, m_H is the Higgs boson mass, $\mathbf{k_{1,2}}$ are the transverse momenta of the colliding gluons and $x_{1,2}$ are their longitudinal momentum fractions (see eq. (2)). The cross-section σ_0 is given by eq. (4). From eqs. (2,3) we also have

$$\tau s = Q^2 + p_T^2 \tag{21}$$

Substituting in eq. (20) the following representation of the δ function,

$$\delta^{(2)}(\mathbf{k_1} + \mathbf{k_2} - \mathbf{p_T}) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b} \, \exp\{-i(\mathbf{k_1} + \mathbf{k_2} - \mathbf{p_T})\mathbf{b}\}, \tag{22}$$

we find that the integral defining the differential cross section for Higgs production can be directly expressed in terms of the unintegrated gluon distributions $\bar{f}_G(x, b, Q)$ in the impact parameter representation

$$\frac{\partial \sigma}{\partial Q^2 \partial y \partial p_T^2} = \frac{\sigma_0 Q^2}{\tau s} \pi^2 \delta(Q^2 - m_H^2) \int_0^\infty db^2 \, \bar{f}_G(x_1, b, Q) \, \bar{f}_G(x_2, b, Q) \, J_0(b p_T). \tag{23}$$

where

$$\bar{f}_G(x, b, Q) = \int_0^\infty dk_T k_T \, J_0(b k_T) \, f_G(x, k_T, b). \tag{24}$$

The cross section for Higgs production which is driven by the $gg \to H$ fusion is determined by the gluon distributions. Similar representation of the cross section, but in terms of the quark and antiquark distributions in the b space, can be obtained in the case of the Drell-Yan production of the muon pairs. The latter reaction reflects the process $q\bar{q} \to \gamma^* \to \mu^+\mu^-$, and the corresponding expression for the cross-section reads

$$\frac{\partial \sigma}{\partial Q^2 \partial y \partial p_T^2} = \frac{\sigma_0^{DY}}{\tau} \int db^2 J_0(b p_T) \sum_i e_i^2 \left[\bar{f}_{q_i}^1(x_1, b, Q) \, \bar{f}_{\bar{q}_i}^2(x_2, b, Q) + \bar{f}_{q_i}^2(x_2, b, Q) \, \bar{f}_{\bar{q}_i}^1(x_1, b, Q) \right] \tag{25}$$

where now p_T, Q and y denote the transverse momentum, invariant mass and rapidity of the lepton pair, respectively, $x_{1,2}$ denote the momentum fractions (2) of the parent hadrons carried by a quark or antiquark. The functions $\bar{f}_{q_i}^{1,2}$ and $\bar{f}_{\bar{q}_i}^{1,2}$ are the quark and antiquark unintegrated distributions in the $b-$representation. As in the case for the gluons, they are linked to the scale dependent k_T distributions of quarks and antiquarks through the Fourier–Bessel transform (cf. eq. (24)). The elementary DY cross section σ_0^{DY} is given by eq. (6). Comparing eqs. (23, 25) with eqs. (1, 5), we find that within the unintegrated parton distrtibution framework the functions $W_{ij}(x_1, x_2, b, Q)$ are given by:

$$W_{ij}(x_1, x_2, b, Q) = 4 \bar{f}_i(x_1, b, Q) \, \bar{f}_j(x_2, b, Q). \tag{26}$$

4 Single loop CCFM equation in the $b-$representation

In this section we recall the CCFM equation in the single loop approximation. Then we extend this equation to the system of equations incorporating unintegrated quark and antiquark distributions, f_{q_i} and $f_{\bar{q}_i}$. The original Catani-Ciafaloni-Fiorani-Marchesini (CCFM) equation

[28, 29] for the unintegrated, scale-dependent gluon distribution $f_G(x, k_T, Q)$ has the following form:

$$f_G(x, k_T, Q) = f_G^0(x, k_T, Q) + \int \frac{d^2\boldsymbol{q}}{\pi q^2} \int_x^1 \frac{dz}{z} \, \Theta(Q - qz) \, \Theta(q - q_0) \frac{\alpha_s}{2\pi} \Delta_S(Q, q, z)$$

$$\times \left[2N_c \, \Delta_{NS}(k_T, q, z) + \frac{2N_c \, z}{1 - z} \right] f\left(\frac{x}{z}, |\mathbf{k_T} + (1 - z)\boldsymbol{q}|, q \right), \tag{27}$$

where $\Delta_S(Q, q, z)$ and $\Delta_{NS}(k_T, q, z)$ are the Sudakov and non-Sudakov form factors,

$$\Delta_S(Q, q, z) = \exp \left\{ - \int_{(qz)^2}^{Q^2} \frac{dp^2}{p^2} \frac{\alpha_s}{2\pi} \int_0^{1 - q_0/p} dz \, z \, P_{gg}(z) \right\} \tag{28}$$

$$\Delta_{NS}(k_T, q, z) = \exp \left\{ - \int_z^1 \frac{dz'}{z'} \int_{(qz')^2}^{k_T^2} \frac{dp^2}{p^2} \frac{2N_c \, \alpha_s}{2\pi} \right\}. \tag{29}$$

The variables x, k_T and Q denote the longitudinal momentum fraction, the transverse momentum of the gluon and the hard scale, respectively. Eq. (27) is generated by the sum of ladder diagrams with angular ordering along the chain. The hard scale Q is defined in terms of the maximum emission angle [26, 28]. The constraint $\Theta(Q - qz)$ in eq. (27) reflects the angular ordering, and the inhomogeneous term, $f^0(x, k_T, Q)$, is related to the input non-perturbative gluon distribution. It also contains effects of both the Sudakov and non-Sudakov form-factors [33].

Eq. (27) interpolates between a part of the DGLAP evolution at large and moderately small values of x, and the BFKL dynamics at small x. To be precise, it contains only the $g \to gg$ splittings and only those parts of the splitting function $P_{gg}(z)$ which are singular at either $z \to 1$ or $z \to 0$. Following [34, 35] we shall make the following extension of the original CCFM equation (27):

1. We introduce, besides the unintegrated gluon distributions $f_G(x, k_T, Q)$, also the uninte-grated quark and antiquark gluon distributions, $f_{q_i}(x, k_T, Q)$ and $f_{\bar{q}_i}(x, k_T, Q)$.

2. We include, in addition to the $g \to gg$ splittings, also the $q \to gq$, $\bar{q} \to g\bar{q}$, and $g \to \bar{q}q$ transitions along the chain.

3. We take into account the complete splitting functions $P_{ab}(z)$, and not only their singular parts.

These extensions make the CCFM framework more accurate in the region of large and moder-ately small values of x (i.e. $x \geq 0.01$ or so). In this region one can also introduce the single-loop approximation that corresponds to the following replacements:

$$\Theta(Q - qz) \longrightarrow \Theta(Q - q)$$
$$\Delta_{NS}(k_T, q, z) \longrightarrow 1. \tag{30}$$

It is also convenient to "unfold" the Sudakov form factor such that the real emission and virtual terms appear on an equal footing in the corresponding evolution equations. Finally, the unfolded system of the extended CCFM equations in the single-loop approximation has the following form:

$$
f_{NS}^i(x, k_T, Q) = f_{NS}^{i0}(x, k_T) + \int_0^1 dz \int \frac{d^2q}{\pi q^2} \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2) \Theta(Q - q)
$$

$$
\times \; P_{qq}(z) \left[\Theta(z - x) f_{NS}^i\left(\frac{x}{z}, k'_T, q\right) - f_{NS}^i(x, k_T, q) \right] \tag{31}
$$

$$
f_S(x, k_T, Q) = f_S^0(x, k_T) + \int_0^1 dz \int \frac{d^2q}{\pi q^2} \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2) \Theta(Q - q)
$$

$$
\times \; \left\{ \Theta(z - x) \left[P_{qq}(z) f_S\left(\frac{x}{z}, k'_T, q\right) + P_{qg}(z) f_G\left(\frac{x}{z}, k'_T, q\right) \right] \right.
$$

$$
\left. - \; P_{qq}(z) f_S(x, k_T, q) \right\} \tag{32}
$$

$$
f_G(x, k_T, Q) = f_G^0(x, k_T) + \int_0^1 dz \int \frac{d^2q}{\pi q^2} \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2) \Theta(Q - q)
$$

$$
\times \; \left\{ \Theta(z - x) \left[P_{gq}(z) f_S\left(\frac{x}{z}, k'_T, q\right) + P_{gg}(z) f_G\left(\frac{x}{z}, k'_T, q\right) \right] \right.
$$

$$
\left. - \; \left[z P_{gg}(z) + z P_{qg}(z) \right] f_G(x, k_T, q) \right\} \tag{33}
$$

where

$$
\mathbf{k'_T} = \mathbf{k_T} + (1 - z)\mathbf{q}. \tag{34}
$$

The functions $f_{NS}^i(x, k_T, Q)$ are the unintegrated non-singlet quark distributions, while the unintegrated singlet distribution, $f_S(x, k_T, Q)$, is defined as

$$
f_S(x, k_T, Q) = \sum_{i=1}^{N_f} \left[f_{q_i}(x, k_T, Q) + f_{\bar{q}_i}(x, k_T, Q) \right] . \tag{35}
$$

The functions $P_{ab}(z)$ are the LO splitting functions corresponding to real emissions, *i.e.*:

$$
\begin{aligned}
P_{qq}(z) &= C_F \frac{1 + z^2}{1 - z} \\
P_{qg}(z) &= N_f \left[z^2 + (1 - z)^2 \right] \\
P_{gq}(z) &= C_F \frac{1 + (1 - z)^2}{z} \\
P_{gg}(z) &= 2N_c \left[\frac{z}{1 - z} + \frac{1 - z}{z} + z(1 - z) \right]
\end{aligned} \tag{36}
$$

where N_f and N_c denote the number of flavours and colours, respectively, and $C_F = (N_c^2 - 1)/(2N_c)$. It should be observed that after integrating both sides of eqs. (31)-(33) over $d^2\mathbf{k_T}$, we get the usual DGLAP equations for the integrated parton distributions $p_k(x, Q^2)$, defined by

$$xp_k(x, Q^2) = \int_0^\infty dk_T^2 \, f_k(x, k_T, Q). \tag{37}$$

A very important merit of eqs. (31)-(33) is the fact that they can be diagonalised by the Fourier-Bessel transform

$$f_k(x, k_T, Q) = \int_0^\infty db \, b \, J_0(bk_T) \, \bar{f}_k(x, b, Q), \tag{38}$$

where the inverse relation reads

$$\bar{f}_k(x, b, Q) = \int_0^\infty dk_T \, k_T \, J_0(bk_T) \, f_k(x, k_T, Q). \tag{39}$$

In the above $k = NS, S, G$, and J_0 is the Bessel function. At $b = 0$ the functions $\bar{f}_k(x, b, Q)$ are related to the integrated distributions $p_i(x, Q^2)$

$$\bar{f}_k(x, b = 0, Q) = \frac{1}{2} xp_k(x, Q^2). \tag{40}$$

The corresponding evolution equations for $\bar{f}_{NS}$, $\bar{f}_S$ and $\bar{f}_G$, which follow from eqs. (31)-(33), have the following form:

$$Q^2 \frac{\partial \bar{f}_{NS}(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 dz \, P_{qq}(z) \left[\Theta(z - x) \, J_0((1 - z)Qb) \, \bar{f}_{NS}\left(\frac{x}{z}, b, Q\right) \right.$$

$$\left. - \, zP_{qq}(z) \, \bar{f}_{NS}(x, b, Q) \right] \tag{41}$$

$$Q^2 \frac{\partial \bar{f}_S(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 dz \left\{ \Theta(z - x) \, J_0((1 - z)Qb) \left[P_{qq}(z) \, \bar{f}_S\left(\frac{x}{z}, b, Q\right) \right. \right.$$

$$\left. \left. + \, P_{qg}(z) \, \bar{f}_G\left(\frac{x}{z}, b, Q\right) \right] - [zP_{qq}(z) + zP_{gq}(z)] \, \bar{f}_S(x, b, Q) \right\} \tag{42}$$

$$Q^2 \frac{\partial \bar{f}_G(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 dz \left\{ \Theta(z - x) \, J_0((1 - z)Qb) \left[P_{gq}(z) \, \bar{f}_S\left(\frac{x}{z}, b, Q\right) \right. \right.$$

$$\left. \left. + \, P_{gg}(z) \, \bar{f}_G\left(\frac{x}{z}, b, Q\right) \right] - [zP_{gg}(z) + zP_{qg}(z)] \, \bar{f}_G(x, b, Q) \right\} \tag{43}$$

with the initial conditions

$$\bar{f}_k(x, b, Q_0) = \bar{f}_k^0(x, b). \tag{44}$$

9

We have therefore found that the CCFM scheme in the single loop approximation does directly provide the system of evolution equations for the unintegrated parton distributions $\bar{f}_k(x, b, Q)$ in the b−representations. These functions are the basic quantities in the description of p_T spectra, eqs. (5),(23), within the unintegrated parton distributions framework. In the next section we discuss their solution and show that they correctly incorporate the resummation effects.

5   Soft gluon resummation formulas from the solution of the CCFM equations

In order to understand the structure of the solution of the CCFM equations (41)-(43), it is convenient to introduce the moment functions $\tilde{f}_i(n, b, Q)$

$$\tilde{f}_i(n, b, Q) = \int_0^1 dx\, x^{n-1} \bar{f}_i(x, b, Q) \tag{45}$$

We shall consider at first the non-singlet, valence quark distributions for which the CCFM equation in the moment space reads:

$$Q^2 \frac{\partial \tilde{f}_{NS}(n, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 dz\, P_{qq}(z) \left\{ z^n J_0((1 - z)Qb) - 1 \right\} \tilde{f}_{NS}(n, b, Q) \tag{46}$$

Its solution reads:
$$\tilde{f}_{NS}(n, b, Q) = \tilde{f}_{NS}(n, b, Q_0) \exp\{I(n, b, Q)\} \tag{47}$$

where the function $I(n, b, Q)$ is given by:

$$I(n, b, Q) = \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz\, P_{qq}(z) \left\{ z^n J_0((1 - z)Qb) - 1 \right\}, \tag{48}$$

and the function $\bar{f}_{NS}(n, b, Q_0)$ is given by the input unintegrated distribution at the reference scale Q_0. At $b = 0$ we have:

$$\tilde{f}_{NS}(n, b = 0, Q) = \frac{1}{2} p_{NS}(n, Q^2) \tag{49}$$

$$\tilde{f}_{NS}(n, b = 0, Q_0) = \frac{1}{2} p_{NS}(n, Q_0^2) \tag{50}$$

where $p(n, Q^2)$ are the moment functions of the integrated distributions. We shall now rearrange the integral $I(n, b, Q)$ as below:

$$I(n, b, Q) = I_1(n, b, Q) + I_2(b, Q) \tag{51}$$

10

where

$$I_1(n, b, Q) = \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz \, P_{qq}(z) \, (z^n - 1) \, J_0((1-z)Qb) \tag{52}$$

$$I_2(b, Q) = \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz \, P_{qq}(z) \, \{J_0((1-z)Qb) - 1\} \, . \tag{53}$$

The dominant contribution to the integrals I_1 and I_2 comes from the region in which argument of the Bessel function $J_0(u)$ is small, where the Bessel function itself can be approximated by unity, i.e. we can adopt the following approximation:

$$J_0(u) \simeq \Theta(c - u) \tag{54}$$

with c being of the order of 1; for simplicity we set $c = 1$. The dominant contribution to the integral I_1 comes from the region $Qb < 1$ and adopting approximation (54) we get for $b > 1/Q$:

$$I_1(n, b, Q) \simeq \int_{Q_0^2}^{1/b^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz \, P_{qq}(z) \, (z^n - 1) \, . \tag{55}$$

We observe that approximate representation (55) of $I_1(n, b, Q)$ is equal to the argument of the exponent describing evolution of the moments of the *integrated* non-singlet quark distributions from the scale Q_0^2 to the scale $1/b^2$. For $b \ll 1/Q_0$ we may set

$$\tilde{f}_{NS}(n, b, Q_0) \simeq \tilde{f}_{NS}(n, b = 0, Q_0) = \frac{1}{2} \tilde{p}_{NS}(n, Q_0^2) \tag{56}$$

to finally get

$$\tilde{f}_{NS}(n, b, Q_0) \, \exp\{I_1(n, b, Q)\} \simeq \frac{1}{2} \tilde{p}_{NS}\left(n, \frac{1}{b^2}\right) \, . \tag{57}$$

After inverting moments one finds

$$\bar{f}_{NS}(x, b, Q) \simeq \frac{1}{2} x \, p_{NS}(x, 1/b^2) \, . \tag{58}$$

Comparing (26) with (7) we find that in the functions $W_{ij}(x_1, x_2, b, Q)$ defined by the non-singlet quark distributions, we have identified the first factor in the soft gluon resummation formula to be equal to the product of the integrated parton distributions at the scale of the order of $1/b^2$.

We now show that after using approximation (54) in the evaluation of $I_2(b, Q)$, the form factor $\exp\{2I_2(b, Q)\}$ becomes equal to the form factor $\exp\{S_q(b, Q)\}$ in the LO approximation with the exponent $S_q(b, Q)$ defined by eq. (9). To this aim we note that after rearranging the splitting function $P_{qq}(z)$

$$P_{qq}(z) = C_F \left[\frac{2}{1 - z} - (1 + z) \right] \, , \tag{59}$$

11

we can represent the integral $I_2(b, Q)$ as the sum of two terms:

$$I_2(b, Q) = I_2^1(b, Q) + I_2^2(b, Q) \qquad (60)$$

where

$$I_2^1(b, Q) = 2\, C_F \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 \frac{dz}{1-z} \left[J_0((1-z)qb) - 1 \right] \qquad (61)$$

$$I_2^2(b, Q) = -C_F \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz\, (1+z) \left[J_0((1-z)qb) - 1 \right] . \qquad (62)$$

Using approximation (54) we get:

$$I_2^1(b, Q) \simeq -\Theta(Q^2 - 1/b^2)\, C_F \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \ln(q^2 b^2) \qquad (63)$$

$$I_2^2(b, Q) \simeq \Theta(Q^2 - 1/b^2)\, C_F \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \left(1 - \frac{1}{bq}\right)\left(1 + \frac{1}{2}\left(1 - \frac{1}{bq}\right)\right)$$

$$\simeq \Theta(Q^2 - 1/b^2)\, \frac{3}{2} C_F \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} . \qquad (64)$$

Thus (60) becomes

$$2\, I_2(b, q) \simeq -2\, \Theta(Q^2 - 1/b^2)\, C_F \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \left[\ln(q^2 b^2) - \frac{3}{2} \right] . \qquad (65)$$

We note that this expression coincides with that given by eq. (9) defining the exponent $S_q(b, Q)$ in the LO approximation. To be precise, the first term corresponding to $I_2^1(b, Q)$ is identically equal to the term defined by $A_q(\alpha_s)$ for the fixed coupling only and there are subleading differences due to different scale of the QCD coupling. It may be observed, however, that we would get exactly the same result for this term choosing the scale of the coupling equal to $(q(1-z))^2$ instead of q^2 in the integrals defining $I_2^1(b, Q)$. The second term corresponding to $I_2^2(b, Q)$ is identical to that defined by $B_q(\alpha_s)$. Finally we observe that the non-perturbative contribution factorises as the Q independent factor, see eq. (47).

The valence quarks dominate in the production of DY muon pairs and in the production of gauge $W^{\mp}$ or Z^0 bosons in $p\bar{p}$ collisions. However, inclusive production of the Higgs bosons, which is dominated by the gluon-gluon fusion, requires the knowledge of gluon and sea quark distributions. We shall now derive the soft gluon resummation formula for the unintegrated gluon and sea quark distributions, and for the function $W_{GG}(x_1, x_2, b, Q)$, starting from the system of the CCFM equations (42,43). In this case we cannot solve the system of the CCFM equations in an analytic form and the soft gluon resummation expressions will follow from the

approximate treatment of the CCFM equations themselves. To this aim it is convenient to integrate these equations and represent them as the system of integral equations:

$$\bar{f}_S(x, b, Q) = \bar{f}_S^0(x, b) + \int_0^1 dz \int \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2) \Theta(Q - q)$$

$$\times \left\{ \Theta(z - x) J_0((1 - z)qb) \left[P_{qq}(z) \bar{f}_S\left(\frac{x}{z}, b, q\right) + P_{qg}(z) \bar{f}_G\left(\frac{x}{z}, b, q\right) \right] \right.$$

$$\left. - [zP_{qq}(z) + zP_{gq}(z)] \bar{f}_S(x, b, q) \right\} \tag{66}$$

$$\bar{f}_G(x, b, Q) = \bar{f}_G^0(x, b) + \int_0^1 dz \int \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2) \Theta(Q - q)$$

$$\times \left\{ \Theta(z - x) J_0((1 - z)qb) \left[P_{gq}(z) \bar{f}_S\left(\frac{x}{z}, b, q\right) + P_{gg}(z) \bar{f}_G\left(\frac{x}{z}, b, q\right) \right] \right.$$

$$\left. - [zP_{gg}(z) + zP_{qg}(z)] \bar{f}_G(x, b, q) \right\}. \tag{67}$$

We rearrange these equations by adding and subtracting in the integrand of (66) the term

$$\frac{\alpha_s(q^2)}{2\pi q^2} [zP_{qq}(z) + zP_{gq}(z)] J_0((1 - z)bq) \bar{f}_S(x, b, q),$$

and similarly in the integrand of (67) the term

$$\frac{\alpha_s(q^2)}{2\pi q^2} [zP_{gg}(z) + zP_{qg}(z)] J_0((1 - z)bq) \bar{f}_G(x, b, q).$$

Thus we obtain

$$\bar{f}_S(x, b, Q) = \bar{f}_S^0(x, b) + \int_0^1 dz \int \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2) \Theta(Q - q)$$

$$\times \left\{ J_0((1 - z)qb) \left[\Theta(z - x) \left[P_{qq}(z) \bar{f}_S\left(\frac{x}{z}, b, q\right) + P_{qg}(z) \bar{f}_G\left(\frac{x}{z}, b, q\right) \right] \right. \right.$$

$$\left. - [zP_{qq}(z) + zP_{gq}(z)] \bar{f}_S(x, b, q) \right]$$

$$\left. - [zP_{qq}(z) + zP_{gq}(z)] [1 - J_0((1 - z)qb)] \bar{f}_S(x, b, q) \right\} \tag{68}$$

13

$$\bar{f}_G(x,b,Q) = \bar{f}_G^0(x,b) + \int_0^1 dz \int \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2) \Theta(Q - q)$$

$$\times \left\{ J_0((1-z)qb) \left[\Theta(z-x) \left[P_{gq}(z) \bar{f}_S\left(\frac{x}{z}, b, q\right) + P_{gg}(z) \bar{f}_G\left(\frac{x}{z}, b, q\right) \right] \right. \right.$$

$$\left. - [zP_{gg}(z) + zP_{qg}(z)] \bar{f}_G(x, b, q) \right]$$

$$\left. - [zP_{gg}(z) + zP_{qg}(z)] [1 - J_0((1-z)qb)] \bar{f}_G(x, b, q) \right\} \tag{69}$$

We consider this equation in the region $b \ll 1/Q_0$, set $b = 0$ in the inhomogeneous term, make the approximation $\{1 - J_0((1-z)qb))\} \simeq \Theta((1-z)qb - 1)$ in the last terms and limit the integration to $q < min\{Q, 1/b\}$ in the remaining terms, to get for $b > 1/Q$:

$$\bar{f}_S(x,b,Q) = \frac{1}{2} xq_S(x, Q_0^2) + \int \frac{dq^2}{q^2} \int_0^1 dz \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2)\left\{ \Theta(1/b - q) \right.$$

$$\times \left[\Theta(z-x) \left[P_{qq}(z) \bar{f}_S\left(\frac{x}{z}, b, q\right) + P_{qg}(z) \bar{f}_G\left(\frac{x}{z}, b, q\right) \right] \right.$$

$$\left. - [zP_{qq}(z) + zP_{gq}(z)] \bar{f}_S(x, b, q) \right]$$

$$\left. - \Theta(Q-q) \Theta(q - 1/b) \Theta(1 - 1/(qb) - z) [zP_{qq}(z) + zP_{gq}(z)] \bar{f}_S(x, b, q) \right\} \tag{70}$$

$$\bar{f}_G(x,b,Q) = \frac{1}{2} xg(x, Q_0^2) + \int \frac{dq^2}{q^2} \int_0^1 dz \frac{\alpha_s(q^2)}{2\pi} \Theta(q^2 - q_0^2)\left\{ \Theta(1/b - q) \right.$$

$$\times \left[\Theta(z-x) \left[P_{gq}(z) \bar{f}_S\left(\frac{x}{z}, b, q\right) + P_{gg}(z) \bar{f}_G\left(\frac{x}{z}, b, q\right) \right] \right.$$

$$\left. - [zP_{gg}(z) + zP_{qg}(z)] \bar{f}_G(x, b, q) \right]$$

$$\left. - \Theta(Q-q) \Theta(q - 1/b) \Theta(1 - 1/(qb) - z) [zP_{gg}(z) + zP_{qg}(z)] \bar{f}_G(x, b, q) \right\} \tag{71}$$

where

$$q_S(x, Q_0^2) = \sum_{i=1}^{N_f} (q_i(x, Q_0^2) + \bar{q}_i(x, Q_0^2)). \tag{72}$$

We note that $f_i(x, b, q)$ and $f_i(x/z, b, q)$ in all terms, except those in the last terms of eqs. (70) and (71), are in the region $q < 1/b$. We shall show that those terms give $1/2\, xq_S(x, 1/b^2)$ and

14

$1/2\, xg(x, 1/b^2)$, respectively. Let us observe at first that for $Q < 1/b$ eqs. (70) and (71) read:

$$\bar{f}_S(x, b, Q) = \frac{1}{2}\, xq_S(x, Q_0^2) + \int_0^1 dz \int \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi}\, \Theta(q^2 - q_0^2)\, \Theta(Q - q)$$

$$\times \left\{ \Theta(z - x) \left[P_{qq}(z)\, \bar{f}_S\left(\frac{x}{z}, b, q\right) + P_{qg}(z)\, \bar{f}_G\left(\frac{x}{z}, b, q\right) \right] \right.$$

$$\left. - \left[zP_{qq}(z) + zP_{gq}(z) \right] \bar{f}_S(x, b, q) \right\} \tag{73}$$

$$\bar{f}_G(x, b, Q) = \frac{1}{2}\, xg(x, Q_0^2) + \int_0^1 dz \int \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi}\, \Theta(q^2 - q_0^2)\, \Theta(Q - q)$$

$$\times \left\{ \Theta(z - x) \left[P_{gq}(z)\, \bar{f}_S\left(\frac{x}{z}, b, q\right) + P_{gg}(z)\, \bar{f}_G\left(\frac{x}{z}, b, q\right) \right] \right.$$

$$\left. - \left[zP_{gg}(z) + zP_{qg}(z) \right] \bar{f}_G(x, b, q) \right\}, \tag{74}$$

which are just the DGLAP equations in the integral form for $1/2\, xg(x, Q^2)$ and $1/2\, xq_S(x, Q^2)$. We therefore get $f_G(x, b) = 1/2\, xg(x, Q^2)$ for $b < 1/Q$. The first terms in eq. (70) and (71) are the same as in the right hand side of equations (73) and (74) except that instead of $\Theta(Q - q)$ we have $\Theta(1/b - q)$. The first terms in eqs. (70) and (71) reduce to $1/2\, xq_S(x, 1/b^2)$ and $1/2\, xg(x, 1/b^2)$. Therefore, eqs. (70) and (71) read:

$$\bar{f}_S(x, b, Q) = \frac{1}{2}\, xq_S(x, 1/b^2) - \int_0^1 dz \int \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi}$$

$$\times\ \Theta(q - 1/b)\, \Theta(Q - q)\, \Theta(1 - 1/(qb) - z)\, [zP_{qq}(z) + zP_{gq}(z)]\, \bar{f}_S(x, b, q) \tag{75}$$

$$\bar{f}_G(x, b, Q) = \frac{1}{2}\, xg(x, 1/b^2) - \int_0^1 dz \int \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi}$$

$$\times\ \Theta(q - 1/b)\, \Theta(Q - q)\, \Theta(1 - 1/(qb) - z)\, [zP_{gg}(z) + zP_{qg}(z)]\, \bar{f}_G(x, b, q), \tag{76}$$

and their solution read:

$$\bar{f}_S(x, b, Q) = \frac{1}{2}\, \exp\{1/2\, \tilde{S}_q(b, Q)\}\, xq_S(x, 1/b^2) \tag{77}$$

$$\bar{f}_G(x, b, Q) = \frac{1}{2}\, \exp\{1/2\, \tilde{S}_G(b, Q)\}\, xg(x, 1/b^2) \tag{78}$$

with $\tilde{S}_q$ and $\tilde{S}_G$ defined by:

$$\tilde{S}_q(b, Q) = -2 \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^{1 - 1/(bq)} dz\, [zP_{qq}(z) + zP_{gq}(z)] \tag{79}$$

$$\tilde{S}_G(b, Q) = -2 \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^{1 - 1/(bq)} dz\, [zP_{gg}(z) + zP_{qg}(z)] \tag{80}$$

15

Substituting explicit expressions for the splitting functions $P_{ij}(z)$ and setting 1 instead of $1 - 1/(bq)$ as the upper integration limit in the integrals over non-singular at $z = 1$ terms, and using eqs. (13)-(19) we get:

$$\tilde{S}_G(b, Q) \simeq - \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \left[A_G^{(1)} \ln(q^2 b^2) + B_G^{(1)} \right] \tag{81}$$

with $A_G^{(1)}$ and $B_G^{(1)}$ given by eq. (11). We also find that $\tilde{S}_q(b, Q)$ is given by eq. (65). As in the case of non-singlet quarks the first term on the right hand side of eq. (81), with the integrand proportional to $\ln(q^2 b^2)$, gives the result which is given by the term with $A_G(\alpha_s)$ in eq. (9) and the fixed coupling only. Thus, the subleading differences are due to a different scale of the QCD coupling. We would get exactly the same result for this term choosing the scale of α_s equal to $[q(1 - z)]^2$ instead of q^2 in the integrand defining $I_2^1(b, Q)$, eq. (61). The second term on the right hand side of eq. (81) is identical to that defined by $B_G(\alpha_s)$ in eq. (9). As in the case of the valence quark distributions also for the gluon and sea quark distributions the non-perturbative effects will enter as the $Q-$independent factor(s).

6 Numerical results

We have shown in the previous sections that the parton shower described by the CCFM equations in the single loop approximation (41)-(43) generate complete resummation effects in LO, i.e. it automatically resums the double and single "large" logarithms $\ln^2(Q^2 b^2)$ in the region $Q^2 b^2 \gg 1$. In this Section we present numerical results based on the exact (numerical) solution of the CCFM equations in the $b-$representation and compare them with the predictions based on the resummation formulas. In our analysis we assume a factorisable form of the initial conditions (44)

$$\bar{f}_k^0(x, b) = \frac{1}{2} F(b) \, x p_k(x, Q_0^2) \,, \tag{82}$$

assuming for simplicity the same input profile $F(b)$ for quarks and gluons. It is determined by a non-perturbative k_T-distribution at the scale Q_0, and at $b = 0$ we have the normalisation condition $F(0) = 1$. Thus, we assume the Gaussian form of the profile function

$$F(b) = \exp\left\{-b^2/b_0^2\right\} \,, \tag{83}$$

where we set $b_0^2 = 4 \, \text{GeV}^{-2}$. The starting integrated distributions $p_k(x, Q_0^2)$ are taken from the LO GRV analysis [36]. With the factorisable form (82) of the starting quark and gluon distributions, we obtain the corresponding factorisation of the function (26)

$$W_{ij}(x_1, x_2, b, Q) = F^2(b) \tilde{W}_{ij}(x_1, x_2, b, Q) \,, \qquad i, j = q, \bar{q}, g \,. \tag{84}$$

For the soft gluon resummation, the expression for the function $\tilde{W}_{ij}$ takes the following form

$$\tilde{W}_{ij}^{resum}(x_1, x_2, b, Q) = \exp\{1/2 \left[S_i(b, Q) + S_j(b, Q)\right]\} x_1 p_i(x_1, 1/b^2) \, x_2 p_j(x_2, 1/b^2) \,. \tag{85}$$

16

In Fig. 1 we show results for $\tilde{W}_{gg}(x_1, x_2, b, Q = M_H)$ plotted as the function of b^2, calculated for $x_1 = x_2 = M_H/\sqrt{s}$ for $\sqrt{s}$ equal to 1.8 TeV (Fig. 1a) and 14 TeV (Fig. 1b). The Higgs boson mass was assumed to be equal to 115 GeV. We show the results based upon the numerical solution of the CCFM equations (41)-(43) (solid lines) and confront them with the predictions based upon the soft gluon resummation formula with the argument of the exponent $S_G(b, Q)$ given by eq. (81) (dotted lines). We also show the results corresponding to $S_G(b, Q)$ given by the standard eq. (9) (dashed lines). In both latter cases the scale $1/b^2$ was replaced by $Q^2/(b^2Q^2+1)$ that guarantees smooth transition to the region $b^2 < 1/Q^2$. We have also replaced $S_G(b, Q)$ by $S_G^{eff}(b, Q)$, setting $S_G^{eff}(b, Q) = 0$ in the region where $S_G(b, Q) > 0$, and $S_G^{eff}(b, Q) = S_G(b, Q)$ in the region where $S_G(b, Q) < 0$. We can see that the function W_{gg} calculated from the exact solution of the CCFM equation stays most of the time below the predictions based upon the resummation formulas. Only at relatively large values of $b \sim 1$ GeV^{-1}, the resummation formula corresponding to $S_G(b, Q)$ given by eq. (81) leads to smaller magnitude of W_{gg} than that corresponding to the exact solution of the CCFM equation. This difference is caused by a different behaviour of the Sudakov-like form factors $T_g = \exp\{S_G(b, Q)\}$ in all three cases, as can be seen in Fig. 2. In this figure we plot as the solid line the function $\exp\{S_G^{CCFM}(b, Q)\}$ with

$$S_G^{CCFM}(b, Q) = \int_{q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz\, z\, [P_{gg}(z) + P_{qg}(z)] \{J_0((1-z)qb) - 1\}, \quad (86)$$

and compare it with the functions resulting from the soft resummation functions with S_G given by eq. (81) (dotted line) and eq. (9) (dashed line).

In Fig. 3 we show the impact of those differences on the transverse momentum distribution of Higgs bosons produced at $y = 0$, eq. (1). We present the results for $\sqrt{s} = 1.8\ TeV$ and for $\sqrt{s} = 14\ TeV$. We can see that the results based upon the exact solution of the CCFM equation (solid lines) gives smaller magnitude of the cross section at its maximum that those generated by the soft gluon resummation formulas (81) (dotted lines) and (9) (dashed lines). The former tends, however, to generate the longer tail of the p_T distribution.

7   Summary and conclusions

In this paper we have discussed the transverse momentum distributions of the Higgs bosons within the unintegrated distributions framework. We considered the $gg \rightarrow H$ fusion process which is the dominant mechanism of inclusive Higgs boson production. The unintegrated distributions were obtained from the CCFM equations in the single loop approximation which is equivalent to the LO DGLAP evolution for the integrated distributions. We have utilised the fact that the CCFM equations in the single loop approximation can be diagonalised in the b representation. We have shown that the conventional soft gluon resummation formulas are embodied in the solution(s) of the CCFM equations. We have examined possible differences between the exact (numerical) solution of the CCFM equation and their approximate solution leading to conventional soft gluon resummation formulas.

Acknowledgments

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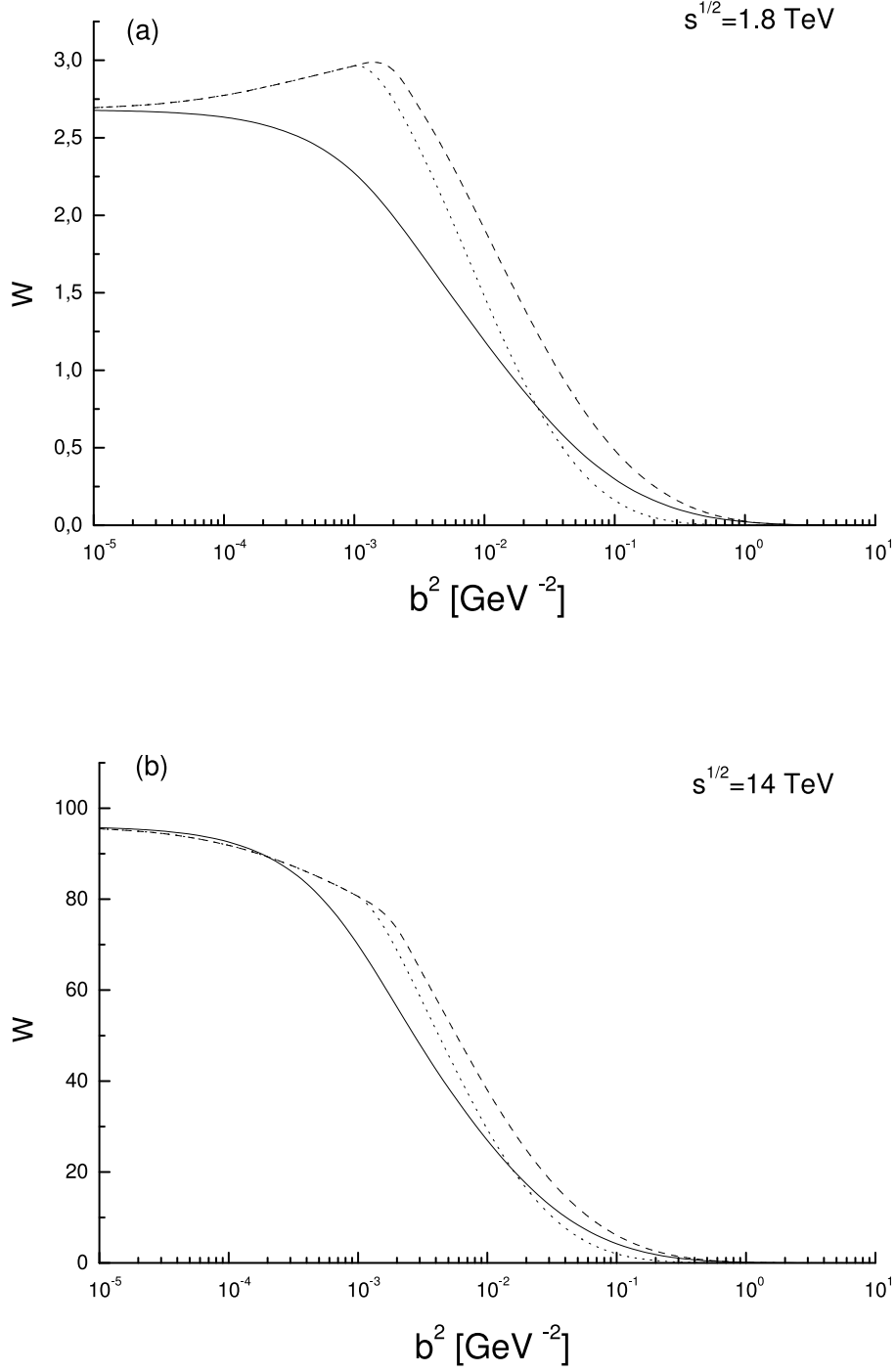[36] M. Glück, E. Reya, A. Vogt, Eur. Phys. J. **C5** (1998) 461.

Figure 1: The function $W_{GG}(x_1, x_2, b, Q)$ as a function of b for the Higgs boson production at $Q = m_H = 115$ GeV in the gluon - gluon fusion at the Tevatron (a) and the LHC energy (b). The solid lines correspond to the result with the unintegrated gluon distributions from the CCFM equations. The results with the LO soft gluon resumation formulas are shown by the dotted lines (S_G given by eq. (81)), and the dashed lines (S_G given by eq. (9)).
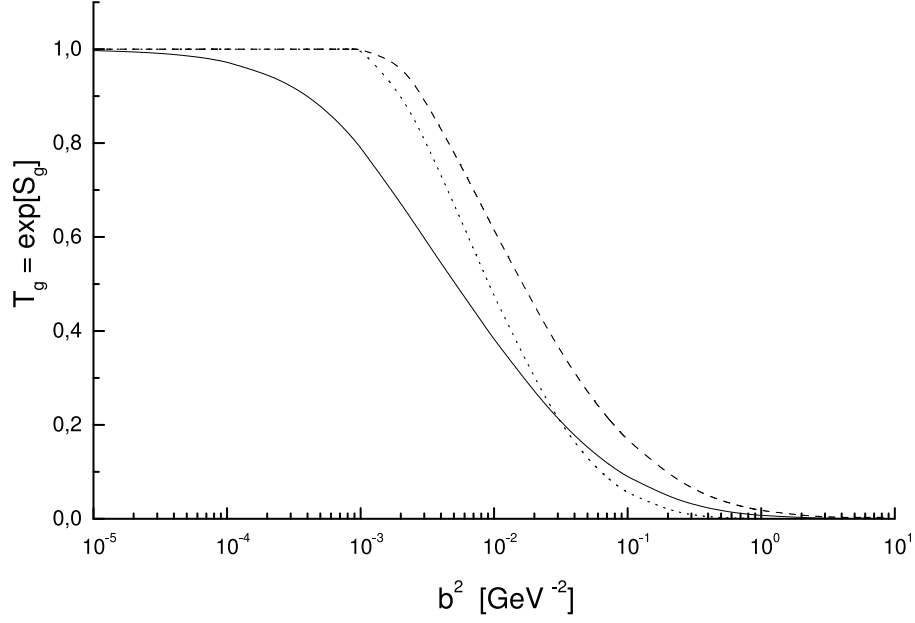
Figure 2: The Sudakow-like form factor $T_G = \exp\{S_G\}$, as a function of the impact parameter b, from the CCFM equations (solid line) and the soft gluon resummation formulas (dotted and dashed lines). The function S_G is given by eq. (86) for the solid line, eq. (81) for the dotted line, and eq. (9) for the dashed line.
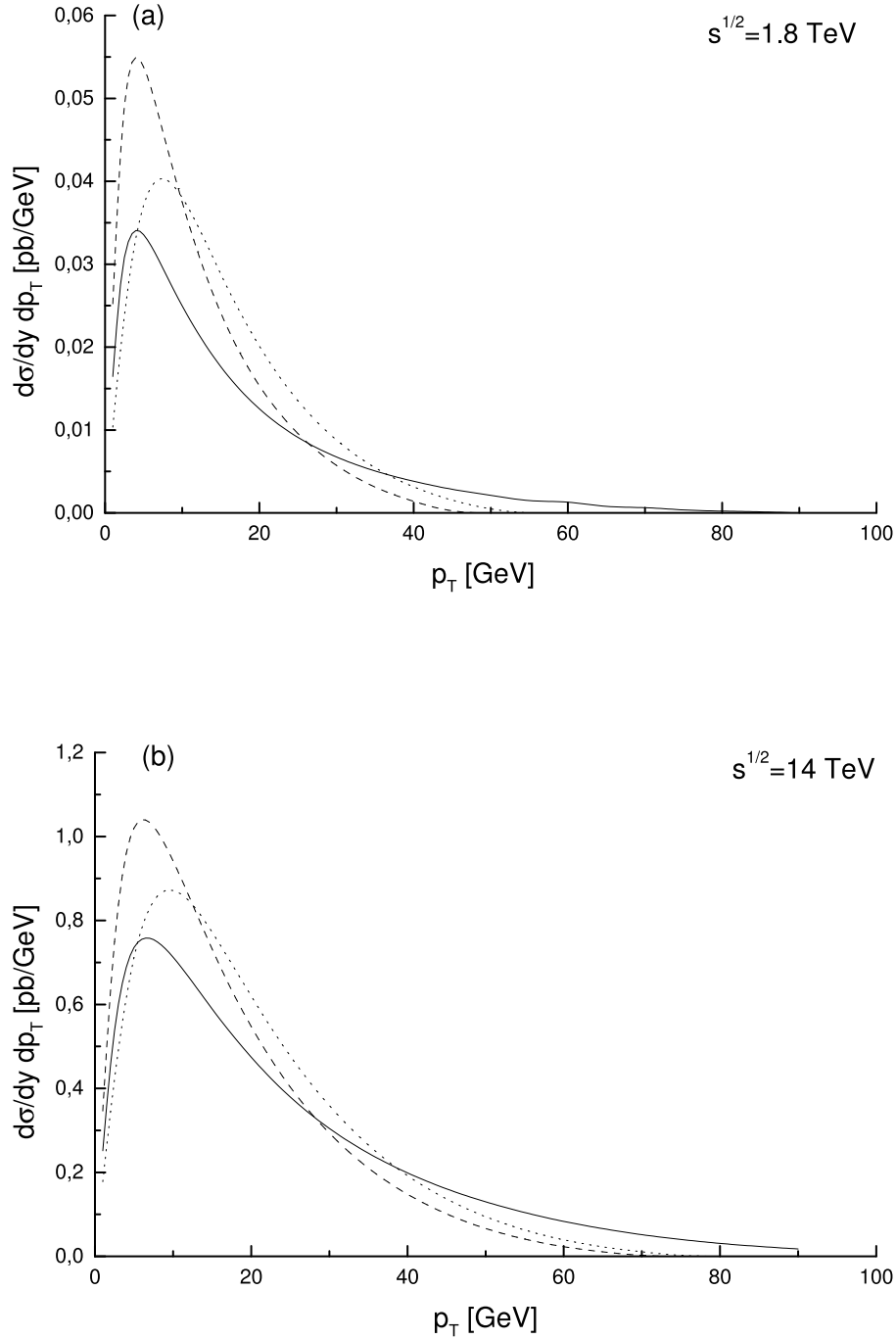
Figure 3: The transverse momentum distribution of Higgs boson from the gluon-gluon fusion at $Q = m_H = 115$ GeV for central rapidity, $y = 0$. Figures (a) and (b) correspond to the Tevatron and LHC energy, respectively. The solid lines are based on the numerical solution of the CCFM equations (41)-(43), while the the dotted and dashed lines results from the LO soft gluon resummation formulas with S_G given by eq. (81) and eq. (9), respectively.